WAVE PROPAGATION IN A NEWTONIAN FLUID WITH VISCOSITY GRADIENTS PROFILES

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INTRODUCTION

Plane compressional wave propagation in a fluid is significantly affected by the shear viscosity of the fluid [1]. Several different theories have been developed [2,3,4] to understand effects of viscosity on wave propagation characteristics Presence of concurring phenomena such as thermal conductive losses and molecular relaxations [5] has further complicated the study of wave propagation In liquids however, the effect of thermal conductivity is not comparable to the viscous losses [1] In such cases, it has been possible to associate viscosity with the wave propagation characteristics

This paper is devoted to an inhomogeneous quasi-static system, where the viscosity is the only contributing parameter to the inhomogeneity. A viscosity gradient in fluids may exist due to various reasons, including temperature gradients, partial reactions and mixing

ASSUMPTIONS

The following simplifying assumptions have been made in developing the governing equation for a media with varying viscosity

1] The principle of superposition is assumed to be valid.
2] The fluid is assumed to follow Newton's law of viscosity.
3] The relation between the bulk and shear viscosities is a constant [6].
4] The viscosity gradient is small (assumed linearity).
5] Viscosity is the only fluid parameter assumed to have a gradient.

DERIVATION

Let there be a viscosity field defined by the function $\eta(x)$. Consider a small volume element *dx.dy.dz* in this field. The stress-strain relationship for this element in tensor notation is given as:

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$$\sigma_{ij} = -\kappa\Delta\delta_{ij} - \frac{2}{3}\eta(x)\frac{\partial\Delta}{\partial t}\delta_{ij} + \eta(x)\left(\frac{\partial^2 u_j}{\partial t\,\partial x_i} + \frac{\partial^2 u_i}{\partial t\,\partial x_j}\right) \tag{1}$$

where σ_{ij} is the stress in the ijth direction, κ is the bulk modulus of the fluid, Δ is the total dilatory strain in the element, δ_{ij} is Kroneckar's delta, $\eta(x)$ is the instantaneous viscosity of the fluid element and u_i is the displacement in the ith direction. A form similar to this Stokesian tensor has been considered for stratified layers [7].

For unidirectional case, Newton's second law of motion is applied to the element. The governing equation of motion along x-direction obtained by neglecting body forces is obtained as:

$$\rho\frac{\partial^2 u}{\partial t^2} = \frac{\partial\sigma_{xx}}{\partial x} + \frac{\partial\sigma_{xy}}{\partial y} + \frac{\partial\sigma_{xz}}{\partial z} \tag{2}$$

Substituting the stress-displacement relation given by equation 1 in equation 2, and considering the propagation of a one-dimensional plane wave along the x axis, simplifies the governing equation further, as the displacements u, v and w remain functions of x and t only. On eliminating the derivatives with respect to y and z, the governing equation reduces to:

$$\rho\frac{\partial^2 u}{\partial t^2} = -\kappa\frac{\partial^2 u}{\partial x^2} + \frac{4}{3}\frac{\partial\eta(x)}{\partial x}\frac{\partial^2 u}{\partial t\,\partial x} + \frac{4}{3}\eta(x)\frac{\partial^3 u}{\partial t\,\partial x^2} \tag{3}$$

Let the viscosity field $\eta(x)$ be continuously differentiable in the field $[0, l]$. The assumed solution to this equation is $e^{i(\omega t - a(x))}$, where $a(x)$, the complex wave number function, varies with x. Then the complex wave function is obtained substituting the assumed solution in the governing equation of motion as:

$$-\rho\omega^2 = -\kappa(ia'(x) - a^2(x)) + \frac{4}{3}\eta'(x)ia(x) + \frac{4}{3}\eta(x)\omega(ia'(x) - a^2(x)) \tag{4}$$

Viscosity is assumed to have a polynomial field variation. The complex wave function is assumed as a series $a(x) = (\kappa_1+i\alpha_1)x + (\kappa_2+i\alpha_2)x^2 + \ldots$ This equation will have multiple eigen values. Two criterions are used to satisfy the physical problem. The media is lossy i e., as the plane wave progresses in the media, it decays with travel distance x, on losing energy. Therefore, α_1 must be negative. The phase depends on the real part of the wave number. Hence κ_1 must be near ω/c_0 at low frequencies and close to $(8\omega\rho^2 c_0^3)/(8\rho^2 c_0^4 + 3\omega^2\eta^2)$ [2] at higher frequencies

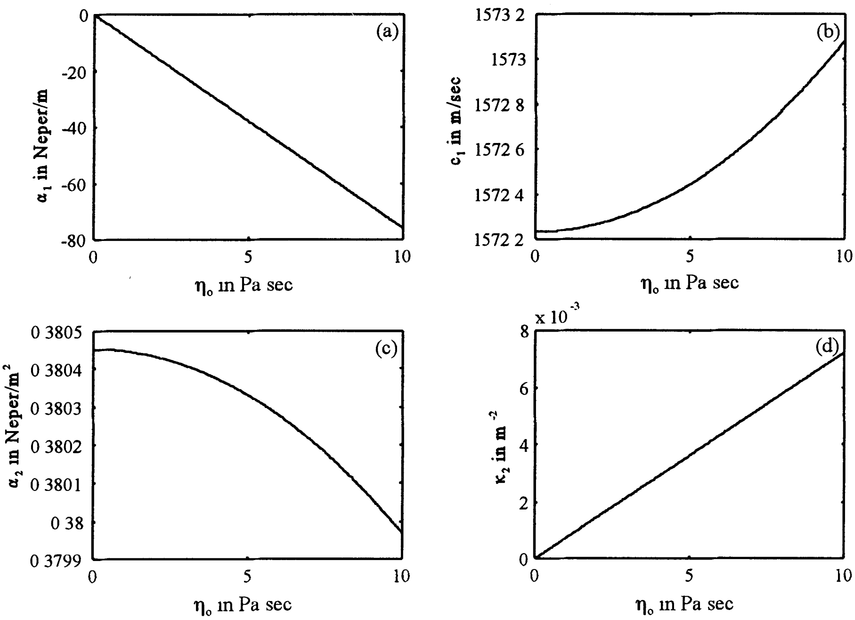

Figure 1 Effect of η_0 on (a) α_1, (b) c_1, (c) α_2 and (d) κ_2 $f = 1$ MHz, K = 2.2 GPa, $\rho = 890$ kg/m^3, and $\eta_1 = 0.1$ Pa sec/m. Linear viscosity field.

ANALYSIS OF WAVE PROPAGATION MODEL

A viscosity field generated by differential temperature (temperature gradient) is essentially exponential in nature. However, for smaller gradients the field could be approximated to a polynomial. Two cases of viscosity field were analyzed, where solutions are feasible Both these cases assume a polynomial viscosity field variation

EFFECT OF LINEAR VISCOSITY FIELD VARIATION

The base viscosity (η_0) is assumed positive, as negative viscosity anywhere in the field runs contrary to the laws of thermodynamics. α_1 responds linearly to η_0 (figure 1a) The wave number (κ_1) also shows a parabolic variation with η_0 (figure 1b). However, dispersion is not significant The attenuation contribution of α_2 decreases with increasing η_0 (figure 2). This may be due to the dominance of the base viscosity that, in turn, is reflected by a larger change in α_1

For a low base viscosity, α_1 does not respond to change in viscosity gradient η_1 The wave speed (c_1) also shows minimal response to change in the viscosity gradient η_1. On the other hand α_2 and κ_2 vary linearly with the viscosity gradient (η_1). This is true in both the cases of positive and negative gradients.

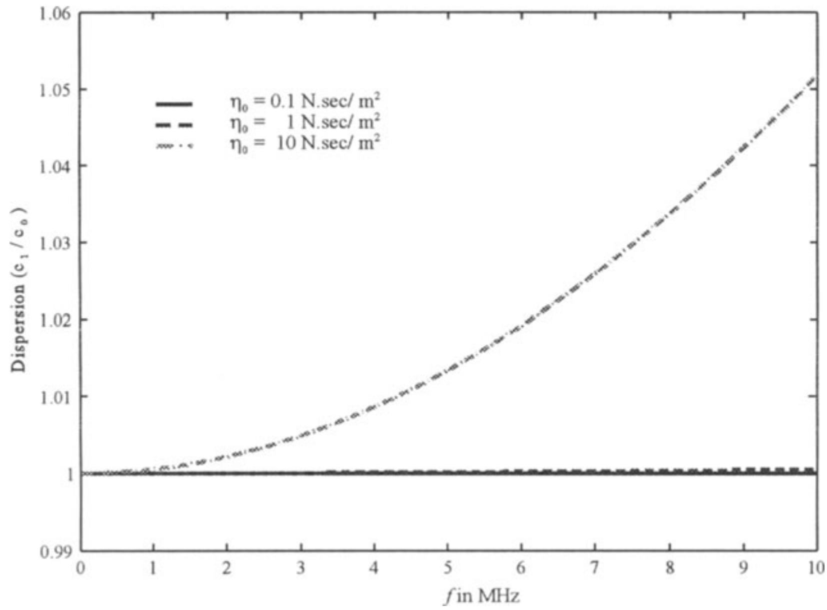

Figure 2 Effect of frequency and η_0 on dispersion. K = 2.2 GPa, ρ = 890 kg/m^3, η_1 = 0 1 Pa sec/m and η_2 = 0 1 Pa.sec/m^2. Quadratic viscosity field

EFFECT OF QUADRATIC VISCOSITY FIELD VARIATION

As in the linear case, α_1 follows a similar relation with η_0 Dispersion (figure 2) characteristics of both the models are observed to be the same, reflecting the independence of κ_1 from η_2 Unlike the linear case, where α_2 is positive for all values of η_0, here it is negative throughout due to the changing viscosity gradient

EFFECT OF FLUID BULK MODULUS AND DENSITY

Bulk modulus affects the elastic response of a fluid to a propagating wave. An increase in the bulk modulus leads to a higher elastic response and lower dispersion and attenuation (figure 3a-d). This is true in the linear and quadratic models. We observed an inverse relation between the base viscosity and bulk modulus A fluid with low viscosity and low bulk modulus property has the same dispersion as a fluid with high bulk modulus and high viscosity κ_2 and κ_3 increase with increasing bulk modulus whereas, attenuation parameters decrease with an increase in the bulk modulus.

We observed a band of low frequencies at which dispersion is minimum. This frequency band also showed an invariance of other dependent parameters of the complex wave function with frequency. Outside this band dispersion is parabolic.

The fluid density was varied from 800 Kg/m^3 to 1100 Kg/m^3. Density did not show any change in the relation between the viscosity parameters and the complex wave function, except translating these by a constant value.

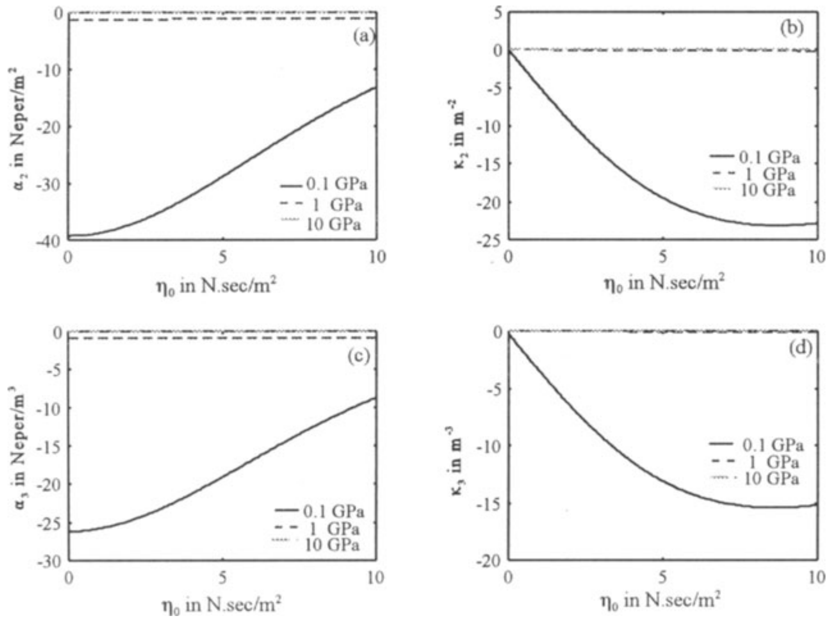

Figure 3. Effect of η_0 and K on (a) α_2, (b) κ_2, (c) α_3 and (d) κ_3. f = 1 MHz, ρ = 890 kg/m^3, η_1 = 0 1 Pa.sec/m and η_2 = 0.1 Pa.sec/m^2 Quadratic viscosity field.

CONCLUSIONS

Compressional stress wave propagation in a fluid with a viscosity gradient is presented. Two types of viscosity fields are considered. In both the cases base viscosity dominates over the viscosity gradients Change in the base viscosity affects all the terms of the complex wave function. Gradients affect only the higher order terms in the complex wave function. The effect of bulk modulus on the complex wave function is dependent on the operating frequency Density does not have a significant effect.

ACKNOWLEDGMENT

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